

The LHC data and an upper bound for the inelastic diffraction.

S.M. Troshin, N.E. Tyurin

*Institute for High Energy Physics,
Protvino, Moscow Region, 142281, Russia*

Abstract

We comment on the status of the Pumplin bound for the inelastic diffraction in the light of the recent LHC data for elastic scattering.

The experiments performed at the LHC have confirmed continuous increase of the total, elastic and inelastic cross-sections with energy, which was observed at lower energies. Those experiments brought us closer to clarification of the elusive asymptotic regime of strong interactions. Arguments based on analyticity and unitarity of the scattering matrix lead to conclusion that the Froissart-Martin bound [1, 2] for the total cross-sections would be saturated at asymptotics. Indeed, the functional energy dependence of the total cross-sections is often taken to follow $\ln^2 s$ -dependence at very high energies, but the value of the factor in front of $\ln^2 s$ remains an issue. This is related to the choice of the upper limit for the partial amplitude. Namely, this limit may correspond to the maximum of the inelastic channel contribution to the elastic unitarity, when

$$\sigma_{el}(s)/\sigma_{tot}(s) \rightarrow 1/2, \quad (1)$$

or it corresponds to a maximal value of the partial amplitude allowed by unitarity resulting in the asymptotical limit

$$\sigma_{el}(s)/\sigma_{tot}(s) \rightarrow 1. \quad (2)$$

The first option is to be an equivalent of the presupposed absorptive nature of the scattering, while the second option assumes the alternative which was interpreted as a reflective scattering [3]. With assumption of the absorptive scattering the original Froissart-Martin bound for the total cross-sections has been improved [4] and the upper bound for the total inelastic cross-section reduced by factor of 4 has also been derived [4].

The assumption on absorptive scattering was also crucial under derivation of the Pumplin bound [5] for the inelastic diffraction:

$$\sigma_{diff}(s, b) \leq \frac{1}{2} \sigma_{tot}(s, b) - \sigma_{el}(s, b), \quad (3)$$

where

$$\sigma_{diff}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{diff}}{db^2}$$

is the total cross-section of all the inelastic diffractive processes in the impact parameter representation and

$$\sigma_{tot}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{tot}}{db^2}, \quad \sigma_{el}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{el}}{db^2}.$$

The inequality Eq. (3) was obtained in the framework of the Good-Walker formalism for the inelastic diffraction [6]. Eq. (3) is to be valid for each value

of the impact parameter of the collision b . It can be integrated over impact parameter with the result

$$\sigma_{diff}(s) \leq \frac{1}{2} \sigma_{tot}(s) - \sigma_{el}(s). \quad (4)$$

Thus, in the framework of the absorptive scattering approach, the Eqs. (1) and (4) should be fulfilled simultaneously if the black disk limit is supposed to be reached asymptotically, i.e.

$$\sigma_{inel}(s)/\sigma_{tot}(s) \rightarrow 1/2 \quad (5)$$

while

$$\sigma_{diff}(s)/\sigma_{tot}(s) \rightarrow 0 \quad (6)$$

and

$$\sigma_{diff}(s)/\sigma_{inel}(s) \rightarrow 0 \quad (7)$$

at $s \rightarrow \infty$. Those limits are the divergent ones. Indeed, $\sigma_{diff}(s)$ ¹ is, by definition², a leading part of the inelastic cross-section $\sigma_{inel}(s)$. The experimental data obtained at the LHC demonstrates approximate energy-independent ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$ [8]. In contrast to the definition of the inelastic diffraction and available experimental data, one should conclude then, that the inelastic diffraction is, in fact, a subleading mechanism of the increase of the inelastic cross-section and the main role in this growth belongs to nondiffractive inelastic processes. Such a statement is not easy to adopt.

There is no such apparent embarrassment in the approach which suppose saturation of the unitarity limit. The assumption that unitarity limit is to be saturated asymptotically leads to a relatively slower increase of the inelastic cross-section

$$\sigma_{inel}(s)/\sigma_{tot}(s) \rightarrow 0 \quad (8)$$

which allows one to keep considering inelastic diffraction as a leading mechanism of the inelastic cross-sections growth. In this approach the ratio of the elastic to total cross-section (2) corresponds to energy increase of the total inelastic cross-section slower than $\ln^2 s$ while Eqs. (2) and (8) take place. It should be noted that available experimental data are consistent with decreasing dependence of the ratio $\sigma_{inel}(s)/\sigma_{tot}(s)$ with energy.

The possibility of the black disk limit crossing was discussed in the general framework of the rational unitarization on the base of the CDF data obtained at Tevatron [9]. It should be noted that the value of $\text{Im}f(s, b=0)$

¹A common opinion associates any type of inelastic diffraction with one or several Pomeron exchanges.

²Cf. for discussion [7].

has increased from 0.36 (CERN ISR) to 0.492 ± 0.008 (Tevatron) and it is on the edge of the black disk limit in this energy domain[10]. As it was mentioned in [9], the exceeding of the black disk limit turns the Pumplin bound to be groundless. But, this conclusion deserves to be more specified now. In fact, the Pumplin bound does not valid only in the limited range of the small and moderate values of the collision impact parameter where absorptive approach is not applicable. We discuss this point here, but we should mention first that the Pumplin bound has been obtained with an assumption of the pure imaginary amplitudes of elastic and diffractive scattering. We use this simplification here.

The model-independent reconstruction of the impact-parameter dependent quantities from this experimental data set demonstrates that the black disk limit has been crossed in elastic scattering at small values of b [11]. In fact, the elastic scattering S -matrix element $S(s, b) \equiv 1 - 2f(s, b)$, where $f(s, b)$ is an imaginary part of the elastic amplitude, is negative at $0 < b < 0.2$ fm and crosses zero at $b = 0.2$ fm at $\sqrt{s} = 7$ TeV. This is consistent with the result [10] of the Tevatron data analysis, in particular. The Pumplin bound can be rewritten in terms of $S(s, b)$ in the form

$$\sigma_{diff}(s, b) \leq \frac{1}{4}S(s, b)(1 - S(s, b)). \quad (9)$$

This inequality clearly indicates that the Pumplin bound cannot be applied in the region where $S(s, b)$ is negative. It should be noted here that this region is determined by the interval $0 < b < R(s)$, where $R(s)$ is the solution of the equation $S(s, b) = 0$. In this impact parameter range only trivial bound

$$\sigma_{diff}(s, b) \leq \sigma_{inel}(s, b)$$

can be applied. But, at $b \geq R(s)$ the scattering is absorptive and therefore the original Pumplin bound should be valid. The integrated bound will be modified, however. Namely, in this case it should be written in the form

$$\bar{\sigma}_{diff}(s) \leq \frac{1}{2}\bar{\sigma}_{tot}(s) - \bar{\sigma}_{el}(s), \quad (10)$$

where $\bar{\sigma}_i(s)$ are the reduced cross-sections:

$$\bar{\sigma}_i(s) \equiv \sigma_i(s) - 8\pi \int_0^{R(s)} b db \sigma_i(s, b),$$

and $i \equiv diff, tot, el$, respectively.

Thus, there is no inconsistency between the saturation of unitarity limit leading to Eq. (2) and the Pumplin bound for the inelastic diffraction cross-section.

References

- [1] M. Froissart, Phys. Rev. **123**, 1053 (1961).
- [2] A. Martin, Nuovo Cimento **42**, 930 (1966).
- [3] S.M. Troshin, N.E. Tyurin, Int. J. Mod. Phys. A **22**, 4437 (2007).
- [4] A. Martin, Phys. Rev. D **80**, 065013 (2009).
- [5] J. Pumplin, Phys. Rev. D **8**, 2899 (1973).
- [6] M.L. Good and W.D. Walker, Phys. Rev. **120**, 1857 (1960) .
- [7] E. Predazzi, Proc. of the Interanational Workshop on Diffraction in High–Energy Physics, Cetraro, Italy, 2-7 September 2000, Eds. R. Fiore, M.I. Kot-sky, A. Papa, E. Predazzi, G. Susinno, Nucl. Phys. B (Proc. Suppl.) **99A**, 3 (2001).
- [8] B. Abelev et al. (ALICE Collaboration), Eur. Phys. J. C **73**, 2456 (2013).
- [9] S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316**, 175 (1993).
- [10] P. Giromini, Proc. of the Vth BLOIS Workshop, Elastic and Diffractive Scat-tering, Eds. H.M. Fried, K. Kang, C-I Tan, World Scientific, Singapore, 1994, p. 30.
- [11] A. Alkin et al., Phys. Rev. D **89**, 091501(R) (2014).